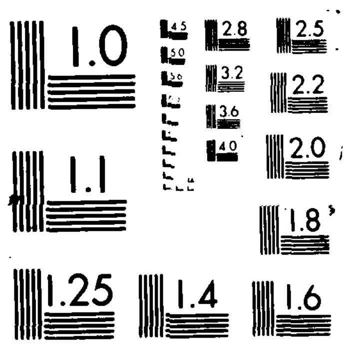
AD-A186 071 PACKET ERROR PROBABILITIES IN FREQUENCY-HOPPED SPREAD SPECTRUM PACKET RAD..(U) VIRGINIA UNIV CHARLOTTESVILLE DEPT OF ELECTRICAL ENGINEERING.. 1/1

UNCLASSIFIED M GEORGIOPOULOS ET AL. 18 SEP 87 F/G 25/2 NL


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

AD-A186 071

DTIC FILE COPY

(12)

Technical Report
Contract No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

PACKET ERROR PROBABILITIES IN FREQUENCY-HOPPED SPREAD SPECTRUM PACKET RADIO NETWORKS - MARKOV FREQUENCY HOPPING PATTERNS CONSIDERED

Submitted to:

Director
Naval Research Laboratory
Washington, D.C. 20375

Attention: Code 2627

Submitted by:

M. Georgiopoulos
EECS Department
University of Central Florida

P. Kazakos
Professor

Report No. UVA/525415/EE88/108

September 1987

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

Technical Report
Contract No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

PACKET ERROR PROBABILITIES IN FREQUENCY-HOPPED SPREAD SPECTRUM PACKET RADIO NETWORKS - MARKOV FREQUENCY HOPPING PATTERNS CONSIDERED

Submitted to:

Director
Naval Research Laboratory
Washington, D.C. 20375

Attention: Code 2627

Submitted by:

M. Georgiopoulos
EECS Department
University of Central Florida

P. Kazakos
Professor

Department of Electrical Engineering
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA

Report No. UVA/525415/EE88/108 Copy No. 15
September 1987

Technical Report
Contract No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

PACKET ERROR PROBABILITIES IN FREQUENCY-HOPPED SPREAD SPECTRUM PACKET RADIO NETWORKS - MARKOV FREQUENCY HOPPING PATTERNS CONSIDERED

Submitted to:

Director
Naval Research Laboratory
Washington, D.C. 20375

Attention: Code 2627

Submitted by:

M. Georgiopoulos
EECS Department
University of Central Florida

P. Kazakos
Professor

Department of Electrical Engineering
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA

Report No. UVA/525415/EE88/108 Copy No. _____
September 1987

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS None
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) UVA/525415/EE88/108	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION University of Virginia Dept. of Electrical Engr. 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Office of Naval Research Resident Representative
6c. ADDRESS (City, State, and ZIP Code) Thornton Hall Charlottesville, VA 22901	7b. ADDRESS (City, State, and ZIP Code) 818 Connecticut Ave., N.W. Eighth Floor Washington, D.C. 20006
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Naval Research 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-86-K-0742
8c. ADDRESS (City, State, and ZIP Code) 800 N. Quincy Street Arlington, VA 22217-5000	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
Packet Error Probabilities in Frequency-Hopped Spread Spectrum Packet Radio Networks-Markov Frequency Hopping Patterns

12. PERSONAL AUTHOR(S)
M. Georgiopoulos/P. Kazakos

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical	FROM 09/01/86 TO 08/31/88	87 September 18	22

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

In this paper we compute the packet error probability induced in a frequency-hopped spread spectrum packet radio network, which utilizes first order Markov frequency hopping patterns. The frequency spectrum is divided into q frequency bins and the packets are divided into M bytes each. Every user in the network sends each of the M bytes of his packet at a frequency bin, which is different from the frequency bin used by the previous byte, but equally likely to be any one of the remaining q-1 frequency bins (Markov frequency hopping patterns). Furthermore, different users in the network utilize statistically independent frequency hopping patterns. Provided that, K users have simultaneously transmitted their packets on the channel, and a receiver has locked on to one of these K packets, we present a method for the computation of

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL . R. N. Madan	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR   83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

$p_e(K)$ (i.e. the probability that this packet is incorrectly decoded). Furthermore, we present numerical results (i.e. $p_e(K)$ versus K) for various values of the multiple access interference K, when Reed Solomon (RS) codes are used for the encoding of packets. Finally, some useful comparisons, with the packet error probability induced, if we assume that the byte errors are independent, are made; based on these comparisons, we can easily evaluate the performance of our spread spectrum system.

TABLE OF CONTENTS

1. Introduction

The rapid growth of computer communication has motivated an intense interest in packet switching radio techniques [1]. Furthermore, there is a growing need for computer communication and information distribution in tactical military applications, where spread spectrum waveforms must be used in order to achieve reliable operation in the precense of intentional interference (jamming). As a result, a thorough investigation of spread spectrum packet radio networks is necessary.

The bit error probability induced in frequency hopped spread spectrum systems has been examined before [2]. In [2] the bit error probability is computed for two different models of random frequency hopping patterns. In the first model it is assumed that each random frequency hopping pattern is a sequence of independent random variables (i.e. memoryless frequency hopping patterns), while in the second model it is assumed that each random frequency hopping pattern is a first order Markov chain (i.e. Markov frequency hopping patterns).

The computation of the packet error probability induced in frequency hopped spread spectrum systems, which utilize memoryless frequency hopping patterns has been examined before ([3],[4],[5]). In this paper we are going to compute the packet error probability induced in spread spectrum systems, which utilize Markov frequency hopping patterns. What makes the problem difficult is that the bit errors are not independent. Hence, we cannot extend the results in [2], in a trivial way, in order to compute the packet error probability. Nevertheless, some comparisons with the packet error probability induced if we assume that the bit errors are independent are going to be made.

The organization of the paper is as follows. In section 2 we describe the model of our spread spectrum system. Then, in section 3 we present a method to compute the packet error probability induced in our system. In section 4 we utilize the numerical results of section 3 and an educated conjecture to evaluate the performance of our spread specrtum system. Finally, in section 5 we give a brief summary of the accomplishments of the paper, and we discuss some extensions of the model presented in section 2.

2. The Model

The frequency hopping system will now be described. The channel time is divided into slots, and the users in the network initiate their packet transmissions at the beginnings of slots. The frequency spectrum is divided into q frequency bins and the packets are divided into M bytes each. Every user in the network sends each of the M bytes of his packet at a frequency bin , which is different from the frequency bin used by the previous byte, but equally likely to be any one of the remaining q-1 frequency bins (Markov frequency hopping patterns). Furthermore, different users in the network have statistically independent frequency hopping patterns. We also assume that a packet consists of exactly one codeword from a Reed-Solomon (RS) code for which up to e byte errors can be corrected. A packet is declared successfully transmitted if at most e byte errors occur.

3. A method to compute the packet error probability

Let us assume that K ($K \geq 2$) packets are transmitted in the same slot. These packets correspond to K different users in the network. We assign indices to these packets (i.e. packet # 1, packet # 2, ... packet # K). Let us also assume that the receiver locks on to packet # 1. We say that the jth byte of packet # 1 is hit if, during its reception by the receiver, at least one of the other packets (i.e. packets # 2, # 3, ..., # K) occupies the same frequency bin that packet # 1 occupies. Let us now denote by ,$p_e(K)$, the probability that packet # 1 is decoded incorrectly by the receiver given that K-1 other packets interfere with packet # 1. Our objective is to provide a method for the computation of $p_e(K)$.

We denote by, { f_j^i ; $1 \leq j \leq M$ }, the frequency hopping pattern corresponding to packet # i ($1 \leq i \leq K$). In figure 1 we show a realization of the K packet arrivals at the receiver site. It is worth noting that the realization of packet arrivals in figure 1 corresponds to the worst possible case; in other words $p_e(K)$ is maximized when the realization of figure 1 occurs. This is true, because with the realization of figure 1 , during the reception of every byte of packet # 1 , all other K-1 interfering packets are present. We will compute $p_e(K)$ for the realization of packet arrivals depicted in figure 1. Furthermore, we will make the pessimistic assumption, as in [3], that when a byte is hit a byte error results.

Let us now denote by, $S(m,n)$; $1 \le n \le M$, $m \le n$, the number of bytes from byte m to byte n of packet # 1 which are in error. Then,

$$p_e(K) = \sum_{i=e+1}^{M} \Pr[S(1,M)=i] \qquad (1)$$

We will compute first the packet error probability , $p_e(K)$, for K=2. In doing so, we will be able to describe the major points of the methodology better. Then, we will discuss the steps required to compute $p_e(K)$ for $K \ge 3$.

<u>Case 1 . K=2.</u>

It is obvious from (1) that, in order to find $p_e(2)$, we must compute the $\Pr[S(1,M)=i]$ for $i=e+1,e+2,\ldots,M$. From the formula of total probability we get:

$$\Pr[S(1,M)=i] = \sum_{s_1^1=1}^{q} \sum_{s_1^2=1}^{q} \Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2) \ \Pr(f_1^1=s_1^1, f_1^2=s_1^2) =$$

$$= \sum_{s_1^1=1}^{q} \sum_{s_1^2=1}^{q} \Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2) \ q^{-2} \qquad (2)$$

We now state a proposition.

<u>Proposition 1.</u> For every $M \ge 1$ and $i \le M$ the conditional probability, $\Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2)$, depends only on the following two events

i) $s_1^1 = s_1^2$ or ii) $s_1^1 \ne s_1^2$

In other words $Pr(S(1,M)=i/f_1^1=s_1^1,f_1^2=s_1^2)$ does not depend on the actual values of s_1^1 and s_1^2, as far as, $s_1^1=s_1^2$ or $s_1^1\neq s_1^2$. Proposition 1 is proven in Appendix A.

Let us now define

$$\underline{f}_j=(f_j^1,f_j^2) \quad 1\leq j\leq M \qquad (3)$$

The proof of Proposition 1 is based on the following Lemma.

Lemma 1. The sequence $\{ \underline{f}_j \ ; \ 1\leq j\leq M \}$ is a Markov chain with stationary transition probabilities.

Lemma 1 is also proven in Appendix A. Due to proposition 1 equation (2) becomes.

$$Pr[S(1,M)=i]= q^{-1}\ Pr(S(1,M)=i/f_1^1=1,f_1^2=1)+$$
$$+(1-q^{-1})\ Pr(S(1,M)=i/f_1^1=1,f_1^2=2) \qquad (4)$$

A byproduct of the proof of proposition 1 is that the conditional probabilities, $Pr(S(1,M)=i/f_1^1=1,f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1,f_1^2=2)$, satisfy certain recurrent expressions (see Appendix A page 16 for more details). We write these recurrent expressions in the sequel.

$$Pr(S(1,1)=0/f_1^1=1,f_1^2=1)=0 \qquad (5a)$$

$$Pr(S(1,1)=1/f_1^1=1,f_1^2=1)=1 \qquad (5b)$$

$$Pr(S(1,1)=0/f_1^1=1,f_1^2=2)=(q-2)/(q-1) \qquad (5c)$$

$$Pr(S(1,1)=1/f_1^1=1,f_1^2=2)=1/(q-1) \qquad (5d)$$

$$Pr(S(1,n)=0/f_1^1=1,f_1^2=1)=0 \qquad (6a)$$

$;2\leq n\leq M$

$$Pr(S(1,n)=i/f_1^1=1,f_1^2=1)=(q-1)^{-1}\ Pr(S(1,n-1)=i-1/f_1^1=1,f_1^2=1)+$$

$;2\leq n\leq M,\ 0<i\leq n$

$$+(q-2)(q-1)^{-1}\ Pr(S(1,n-1)=i-1/f_1^1=1,f_1^2=2) \qquad (6b)$$

$$Pr(S(1,n)=0/f_1^1=1,f_1^2=2)=(q-2)^2(q-1)^{-2}\ Pr(S(1,n-1)=0/f_1^1=1,f_1^2=2) \qquad (7a)$$

$;2\leq n\leq M$

$$Pr(S(1,n)=i/f_1^1=1,f_1^2=2)=(q-2)(q-1)^{-2}\ Pr(S(1,n-1)=i/f_1^1=1,f_1^2=1)+$$

$;2\leq n\leq M,\ 0<i<n$

$$+(q-1)^{-1}\ Pr(S(1,n-1)=i-1/f_1^1=1,f_1^2=2)+$$
$$+(q-2)^2(q-1)^{-2}\ Pr(S(1,n-1)=i/f_1^1=1,f_1^2=2) \qquad (7b)$$

$$Pr(S(1,n)=n/f_1^1=1,f_1^2=2)=(q-1)^{-1}\ Pr(S(1,n-1)=n-1/f_1^1=1,f_1^2=2) \qquad (7c)$$

$;2\leq n\leq M$

Based on expressions (5) through (7) we can compute $Pr(S(1,M)=i/f_1^1=1,f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1,f_1^2=2)$ for arbitrary M and $i\leq M$ as follows.

For M=1 all we need are expressions (5). For M>1 we start from n=2 and we evaluate the probabilities $Pr(S(1,n)=i/f_1^1=1,f_1^2=2)$ and $Pr(S(1,n)=i/f_1^1=1,f_1^2=2)$ for i=0,1,...,n based on expressions (6) and (7). Then, we perform similar computations for n=3,4,... up to n=M. Finally we end up having computed the probabilities $Pr(S(1,M)=i/f_1^1=1,f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1,f_1^2=2)$ for i=0,1,2,.....M.

Once we have computed $Pr(S(1,M)=i/f_1^1=1,f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1,f_1^2=2)$ for i=0,1,2,...,M ,we can find $Pr(S(1,M)=i)$ for i=0,1,2,...,M through formula (4). As a result we can compute $p_e(2)$ via formula (1).

Case 2. $K\geq 3$

The same methodology applies when $K\geq 3$. Hence,

$$\Pr[S(1,M)=i]=\sum_{s_1^1=1}^{q}\cdots\sum_{s_1^K=1}^{q} \Pr(S(1,M)=i/f_1^1=s_1^1,\ldots,f_1^K=s_1^K)\; q^{-K} \quad (8)$$

Furthermore, a proposition and a Lemma, which are similar to proposition 1 and Lemma 1, can also be stated for every $K\geq 3$. What makes the method more complicated, as K increases, is that the number of events, on which $\Pr(S(1,M)=i/f_1^1=s_1^1,\ldots,f_1^K=s_1^K)$ depends on, increases as well. For example, when K=3 the probability $\Pr(S(1,M)=i/f_1^1=s_1^1,f_1^2=s_1^2,f_1^3=s_1^3)$ depends on the following four events:

i) $s_1^1=s_1^2=s_1^3$

or ii) $s_1^1=s_1^2$ and $s_1^1\neq s_1^3$ or $s_1^1\neq s_1^2$ and $s_1^1=s_1^3$

or iii) $s_1^1\neq s_1^2$ and $s_1^1\neq s_1^3$ and $s_1^2=s_1^3$

or iv) $s_1^1\neq s_1^2$ and $s_1^1\neq s_1^3$ and $s_1^2\neq s_1^3$

Nevertheless, once the distinct events, on which the probability $\Pr(S(1,M)=i/f_1^1=s_1^1,\ldots,f_1^K=s_1^K)$ depends on, have been correctly identified, recursive expressions, similar to expressions (5) through (7), can also be written for every $K\geq 3$. Consequently, following the same methodology presented in case 1, one can compute $p_e(K)$ for $K\geq 3$.

Numerical Results.

In Table 1 we have included $p_e(2)$ and $p_e(3)$ for q=10,25 and 50 and for the (31,7),(31,15),(31,23),(63,15),(63,31) and (63,47)

Reed Solomon (RS) codes. In the same table, we have also included $\tilde{p}_e(2)$ and $\tilde{p}_e(3)$, where $\tilde{p}_e(K), K\geq 2$, corresponds to the packet error probability induced by our spread spectrum system under the assumption that the byte errors of packet # 1 are independent. From table 1 we can make the following observations.

1) $\tilde{p}_e(2)$ is an upper bound of $p_e(2)$ and $\tilde{p}_e(3)$ is an upper bound of $p_e(3)$ for most entries of table 1.

2) $|p_e(2)-\tilde{p}_e(2)|/\tilde{p}_e(2)$ and $|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$ are smaller than one for all entries of table 1.

We will see, in the next section, how observation 2 will be helpful in evaluating the performance of our spread specrum system.

Note that if K users are present in a slot then,

$$\tilde{p}_e(K) = \sum_{i=e+1}^{M} \binom{M}{i} [p_b(K)]^i \, [1-p_b(K)]^{M-i} \qquad (10)$$

where $p_b(K)$ is the byte error probability given K. It is easy to show that

$$p_b(K) = 1-(1-2/q)^{K-1} \qquad (11)$$

where q is the number of frequency bins in the frequency spectrum.

4. Performance evaluation of the spread spectrum system.

Suppose that the input packet arival process per slot is Poisson with intensity s. Then, let us define (as in [4])

$$q_e(s)=\sum_{K=0}^{\infty} e^{-s}\ s^K/K!\ p_e(K) \qquad (12)$$

to be the average packet error probability induced by our spread spectrum network at input rate s (note that $p_e(0)=p_e(1)=0$). We also define (as in [4]) the maximum average interference level that can be accomodated at a given packet error probability v by

$$s^*(v)= \max\{s:\ q_e(s)\le v\} \qquad (13)$$

Clearly the value of $s^*(v)$ depends on the number of frequency slots q and the code rate r=k/M (k=number of information bytes, M=total number of bytes in the code). If we account for the expansion of bandwidth induced by q and r, then we can define the normalized maximum average interference level as

$$S^*(v)= r\ s^*(v)/q \qquad (14)$$

Let us also define,

$$\tilde{q}_e(s)=\sum_{K=0}^{\infty} e^{-s}\ s^K/K!\ \tilde{p}_e(K) \qquad (15)$$

(note that $\tilde{p}_e(0)=\tilde{p}_e(1)=0$)

$$\tilde{s}^*(v)= \max\{s:\ \tilde{q}_e(s)\le v\} \qquad (16)$$

and

$$\tilde{S}^*(v)= r\ \tilde{s}^*(v)/q \qquad (17)$$

According to observation 2 of the previous section the ratios $|p_e(2)-\tilde{p}_e(2)|/\tilde{p}_e(2)$ and $|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$ are smaller than one. We can now state a conjecture.

<u>Conjecture</u> : $|p_e(K)-\tilde{p}_e(K)|/\tilde{p}_e(K)\le 1$ for $K\ge 4$.

If this conjecture is true, and we find s such that

$$\tilde{q}_e(s)\ \le\ v \qquad (18)$$

then the same s quarantees that

$$q_e(s) \leq 2v \qquad (19)$$

The above discussion indicates that we can use $\tilde{q}_e(s)$ and $\tilde{s}^*(v)$ as reasonable estimates of the performance of our spread spectrum system, provided that the above conjecture is true. It is worth noting that $\tilde{q}_e(s)$ and $\tilde{s}^*(v)$ are easily computable.

5. Conclusions.

We have described (section 3) a method of computing the packet error probability, $p_e(K)$, induced in a spread spectrum system which utilizes first order Markov frequency hopping patterns. We have also computed $p_e(K)$ for K=2 and K=3. Then, we made some comparisons between $p_e(2)$ and $\tilde{p}_e(2)$ and between $p_e(3)$ and $\tilde{p}_e(3)$; $\tilde{p}_e(K)$, $K \geq 2$, was defined to be the packet error probability induced by our spread spectrum system if we assume that the byte errors are independent. Based on these comparisons and an educated conjecture we have evaluated the performance of our spread spectrum system (section 4).

The consideration of a slotted channel in section 2 is not so restrictive. The methodology of section 3 and the discussion of section 4 are still valid for the unslotted channel, provided that we resort to an upper bound for the packet error probability. Consider the transmission of a given packet in the unslotted channel, and consider a second situation in which it is assumed that the interference level is constant and equal to the maximum number K* of interfering transmissions that take place at any time during the transmission of the given packet in the unslotted system. Clearly, as table 1 also indicates, the packet error probability for the second system is larger than for the first. Hence, $p_e(K^*)$ of table 1 will be an upper bound of the packet error probability of the unslotted system.

Furthermore, the pessimistic assumption that a byte hit results in a byte error need not be made either. More optimistic assumptions described in [5] section IV, where thermal noise is also present, can be incorporated in our model too. They will simply make the presentation of section 3 more complicated.

Appendix A

We will first prove Lemma 1. Let us now denote

$$\underline{f}_j=(f_j^1,f_j^2) \quad ; \ j\geq 1 \qquad (A.1)$$

as in (3). Furthermore, we denote

$$\underline{s}_j=(s_j^1,s_j^2) \quad ; \ j\geq 1 \qquad (A.2)$$

We will show first that

$$\Pr(\underline{f}_j=\underline{s}_j/\underline{f}_{j-1}=\underline{s}_{j-1},\ldots,\underline{f}_1=\underline{s}_1)=\Pr(\underline{f}_j=\underline{s}_j/\underline{f}_{j-1}=\underline{s}_{j-1}) \qquad (A.3)$$

for any $\underline{s}_1,\ldots,\underline{s}_j$ belonging in the state space I (in our case $I=\{\ (i_1,i_2):\ i_1=1,\ldots q,\ \ i_2=1,\ldots q\ \})$ and for any $j\geq 2$.

(A.3) is equivalent to

$$\Pr(\underline{f}_j=\underline{s}_j,\ldots,\underline{f}_1=\underline{s}_1)=\Pr(\underline{f}_{j-1}=\underline{s}_{j-1},\ldots,\underline{f}_1=\underline{s}_1)\ \Pr(\underline{f}_j=\underline{s}_j,\underline{f}_{j-1}=\underline{s}_{j-1})$$
$$[\Pr(\underline{f}_{j-1}=\underline{s}_{j-1})]^{-1} \qquad (A.4)$$

But,

$$\Pr(\underline{f}_j=\underline{s}_j,\ldots,\underline{f}_1=\underline{s}_1)=\Pr(f_j^1=s_j^1,f_j^2=s_j^2,\ldots,f_1^1=s_1^1,f_1^2=s_1^2)=$$

independence of $\{f_j^1;j\geq 1\}$ and $\{f_j^2;j\geq 1\}$

$$=\Pr(f_j^1=s_j^1,\ldots,f_1^1=s_1^1)\ \Pr(f_j^2=s_j^2,\ldots,f_1^2=s_1^2)=$$
$$=\Pr(f_j^1=s_j^1/f_{j-1}^1=s_{j-1}^1,\ldots,f_1^1=s_1^1)\ \Pr(f_{j-1}^1=s_{j-1}^1,\ldots,f_1^1=s_1^1)$$
$$\Pr(f_j^2=s_j^2/f_{j-1}^2=s_{j-1}^2,\ldots,f_1^2=s_1^2)\ \Pr(f_{j-1}^2=s_{j-1}^2,\ldots,f_1^2=s_1^2)=$$

$\{f_j^1;j\geq 1\}$ and $\{f_j^2;j\geq 1\}$ Markov chains

$$=\Pr(f_j^1=s_j^1/f_{j-1}^1=s_{j-1}^1)\ \Pr(f_j^2=s_j^2/f_{j-1}^2=s_{j-1}^2)$$
$$\Pr(f_{j-1}^1=s_{j-1}^1,\ldots,f_1^1=s_1^1)\ \Pr(f_{j-1}^2=s_{j-1}^2,\ldots,f_1^2=s_1^2)=$$

independence of $\{f_j^1;j\geq 1\}$ and $\{f_j^2;j\geq 1\}$

$$=\Pr(\underline{f}_{j-1}=\underline{s}_{j-1},\ldots,\underline{f}_1=\underline{s}_1)\ \Pr(\underline{f}_j=\underline{s}_j,\underline{f}_{j-1}=\underline{s}_{j-1})/\Pr(\underline{f}_{j-1}=\underline{s}_{j-1}) \qquad (A.5)$$

The series of steps in (A.5) prove (A.4). The stationarity of $\{\underline{f}_j;j>1\}$ follows from the fact that

$$\Pr(\underline{f}_j=\underline{s}_j/\underline{f}_{j-1}=\underline{s}_{j-1})=(q-1)^{-2} \quad ;j\geq 2 \qquad (A.6)$$

(A.5) and (A.6) prove the Lemma. We will now prove proposition 1.

Let us first denote by, T1(M), the following statement. "The probability, $Pr(S(1,M)=i/f_1^1=s_1^1,f_1^2=s_1^2)$, with $s_1^1=s_1^2$, is independent of the actual values of s_1^1 and s_1^2, for every i such that $0\le i\le M$."

and by, T2(M), the following statement. "The probability, $Pr(S(1,M)=i/f_1^1=s_1^1,f_1^2=s_1^2)$, with $s_1^1\ne s_1^2$, is independent of the actual values of s_1^1 and s_1^2, for for every i such that $0\le i\le M$."

We will prove, by induction, that T1(M) and T2(M) are true. For M=1 we have:

$$Pr(S(1,1)=0/f_1^1=s_1^1,f_1^2=s_1^2)=0 \quad ;s_1^1=s_1^2 \tag{A.7}$$

$$Pr(S(1,1)=1/f_1^1=s_1^1,f_1^2=s_1^2)=1 \quad ;s_1^1=s_1^2 \tag{A.8}$$

$$Pr(S(1,1)=0/f_1^1=s_1^1,f_1^2=s_1^2)= \quad ;s_1^1\ne s_1^2$$

$$=Pr(f_1^1\ne f_1^2,f_1^1\ne f_2^2/f_1^1=s_1^1,f_1^2=s_1^2)$$

see figure 1

$$=\sum_{\substack{s_2^2=1\\ ;s_2^2\ne s_1^2,\ s_2^2\ne s_1^1}}^{q} Pr(f_1^1\ne f_1^2,f_1^1\ne f_2^2/f_1^1=s_1^1,f_1^2=s_1^2,f_2^2=s_2^2)\ Pr(f_2^2=s_2^2/f_1^1=s_1^1,f_1^2=s_1^2) =$$

$$=(q-2)/(q-1) \tag{A.9}$$

Similarly, we can show that

$$Pr(S(1,1)=1/f_1^1=s_1^1,f_1^2=s_1^2)=1/(q-1) \quad ;s_1^1\neq s_1^2 \qquad (A.10)$$

(A.7),(A.8),(A.9) and (A.10) indicate that T1(1) and T2(1) are true. Suppose that T1(M) and T2(M) are also true for M=n-1. Let us now denote

$$b_1(i)=Pr(S(1,n-1)=i/f_1^1=s_1^1/f_1^2=s_1^2) \quad ;0\leq i\leq n-1,\ s_1^1=s_1^2 \qquad (A.11)$$ and

$$b_2(i)=Pr(S(1,n-1)=i/f_1^1=s_1^1,f_1^2=s_1^2) \quad ;0\leq i\leq n-1,\ s_1^1\neq s_1^2 \qquad (A.12)$$

We will now show that T1(M) and T2(M) are true for M=n. We first note that

$$Pr(S(1,n)=0/f_1^1=s_1^1,f_1^2=s_1^2)=0 \qquad (A.13)$$

$$;s_1^1=s_1^2$$

Furthermore,

$$Pr(S(1,n)=i/f_1^1=s_1^1,f_1^2=s_1^2)=$$

$$;s_1^1=s_1^2,\ 0<i\leq n$$

$$=\sum_{s_2^1=1}^{q}\sum_{s_2^2=1}^{q} Pr(S(1,n)=i/f_1^1=s_1^1,f_1^2=s_1^2,f_2^1=s_2^1,f_2^2=s_2^2)$$

$$Pr(f_2^1=s_2^1,f_2^2=s_2^2/f_1^1=s_1^1,f_1^2=s_1^2)=$$

$$=\sum_{s_2^1=1}^{q}\quad\sum_{s_2^2=1}^{q} Pr(S(1,n)=i/f_1^1=s_1^1,f_1^2=s_1^2,f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}+$$

$$s_2^2=s_2^1 \text{ and } s_2^2\neq s_1^1$$

$$+\sum_{s_2^1=1}^{q}\quad\sum_{s_2^2=1}^{q} Pr(S(1,n)=i/f_1^1=s_1^1,f_1^2=s_1^2,f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}+$$

$$s_2^2=s_2^1 \text{ and } s_2^2=s_1^1$$

$$+\sum_{\substack{s_2^1=1}}^{q}\ \sum_{\substack{s_2^2=1\\ s_2^2\neq s_2^1 \text{ and } s_2^2\neq s_1^1}}^{q} \Pr(S(1,n)=i/f_1^1=s_1^1,f_1^2=s_1^2,f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}+$$

$$+\sum_{\substack{s_2^1=1}}^{q}\ \sum_{\substack{s_2^2=1\\ s_2^2\neq s_2^1 \text{ and } s_2^2=s_1^1}}^{q} \Pr(S(1,n)=i/f_1^1=s_1^1,f_1^2=s_1^2,f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}=$$

(we cannot have $s_1^1=s_1^2$ and $s_2^2=s_1^1$)

$$=\sum_{\substack{s_2^1=1}}^{q}\ \sum_{\substack{s_2^2=1\\ s_2^2=s_2^1 \text{ and } s_2^2\neq s_1^1}}^{q} \Pr(S(2,n)=i-1/f_1^1=s_1^1,f_1^2=s_1^2,f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}+$$

$$+\sum_{\substack{s_2^1=1}}^{q}\ \sum_{\substack{s_2^2=1\\ s_2^2\neq s_2^1 \text{ and } s_2^2\neq s_1^1}}^{q} \Pr(S(2,n)=i-1/f_1^1=s_1^1,f_1^2=s_1^2,f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}=$$

$\{\underline{f}_j;j\geq 1\}$ is a Markov chain (Lemma 1)

$$=\sum_{\substack{s_2^1=1}}^{q}\ \sum_{\substack{s_2^2=1\\ s_2^2=s_2^1 \text{ and } s_2^2\neq s_1^1}}^{q} \Pr(S(2,n)=i-1/f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}+$$

$$+\sum_{\substack{s_2^1=1}}^{q}\ \sum_{\substack{s_2^2=1\\ s_2^2\neq s_2^1 \text{ and } s_2^2\neq s_1^1}}^{q} \Pr(S(2,n)=i-1/f_2^1=s_2^1,f_2^2=s_2^2)\ (q-1)^{-2}=$$

$\{\underline{f}_j;j\geq 1\}$ is stationary (Lemma 1)

$$=\sum_{\substack{s_2^1=1 \\ s_2^2=s_2^1 \text{ and } s_2^2\neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n-1)=i-1/f_1^1=s_2^1, f_1^2=s_2^2)\ (q-1)^{-2}+$$

$$+\sum_{\substack{s_2^1=1 \\ s_2^2\neq s_2^1 \text{ and } s_2^2\neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n-1)=i-1/f_1^1=s_2^1, f_1^2=s_2^2)\ (q-1)^{-2} \qquad \text{(A.14)}$$

We now make the following observations.

O.1) If $\underline{f}_1$ is in state (s_1^1, s_1^2) with $s_1^1=s_1^2$, then $\underline{f}_2$ can be in q-1 different states (s_2^1, s_2^2) with the properties $s_2^2=s_2^1$ and $s_2^2\neq s_1^1$, independently of the actual values of s_1^1 and s_1^2.

O.2) If $\underline{f}_1$ is in state (s_1^1, s_1^2) with $s_1^1=s_1^2$, then $\underline{f}_2$ can be in (q-1)(q-2) different states with the properties $s_2^2\neq s_2^1$ and $s_2^2\neq s_1^1$, independently of the actual values of s_1^1 and s_1^2.

From (A.14), O.1 and O.2 we conclude that

$$\Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2)=b_1(i-1)\ (q-1)/(q-1)^2+$$
$$;s_1^1=s_1^2,\ 0<i\leq n$$
$$+b_2(i-1)\ (q-1)(q-2)/(q-1)^2 \qquad \text{(A.15)}$$

Similarly, we can show that

$$\Pr(S(1,n)=0/f_1^1=s_1^1, f_1^2=s_1^2)=(q-2)^2/(q-1)^2\ b_2(0) \qquad \text{(A.16)}$$
$$;s_1^1\neq s_1^2$$

$$\Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2)=\ (q-2)/(q-1)^2\ b_1(i)+$$
$$;s_1^1\neq s_1^2,\ 0<i<n$$
$$+(q-1)/(q-1)^2\ b_2(i-1)+\ (q-2)^2/(q-1)^2\ b_2(i) \qquad \text{(A.17)}$$

and,

$$Pr(S(1,n)=n/f_1^1=s_1^1,f_1^2=s_1^2)=(q-1)/(q-1)^2\ b_2(n-1) \qquad \text{(A.18)}$$

$$;s_1^1 \neq s_1^2$$

Expressions (A.15) through (A.18) prove that T1(M) and T2(M) are true for M=n as well. Hence, by induction, T1(M) and T2(M) are true for every M≥1.

Furthermore, the induction procedure above and in particular expressions (A.15) trough (A.18) verified the validity of equations (5) through (7) of section 3.

REFERENCES.

[1] R. E. Kahn, S. A. Gronemeyer, J. Burchfiel and R. C. Kunzelman, "Advances in packet radio technology", Proc. IEEE November 1978.

[2] E. A. Geraniotis and M. B. Pursley, "Error probabilities for slow-frequency-hopped spread-spectrum multiple-access communications over fading channels", IEEE Trans. on Comm., Vol. COM-30, pp. 996-1099, May 1982.

[3] B. Hajek, "Recursive retransmission control- Application to a frequency-hopped spread-spectrum system", in Proc. 1982 Conf. Info. Sci. and Systems, Mar 1982, pp. 116-120.

[4] M. B. Pursley, "Frequency-hop transmission for satellite packet switching and terrestrial packet radio networks", IEEE Transactions on IT, Vol. IT-32, No. 5, Sept. 1986.

[5] M. B. Pursley and D. J. Taipale, "Error probabilities for spread-spectrum packet radio with convolutional codes and Viterbi decoding", IEEE Trans. on Comm., Vol. COM-35, No. 1, pp. 1-13, Jan. 1987.

[6] J. E. Wieseltier, and A. Ephremides, "Discrimination against partially overlapping interference- Its effect on throughput in frequency-hopped multiple access channels", IEEE Trans. on Comm., Vol. COM-34, No. 2, pp. 136-142, February 1986.

[7] K. L. Chung, Markov Chains with stationary transitional probabilities, Springer-Verlag, 1960.

[8] R. L. Pickholtz, D. L. Schilling, and L. B. Milstein, "Theory of spread-spectrum communications. A tutorial", IEEE Trans. on Comm., Vol. COM-30, No. 5, pp. 855-884, May 1982.

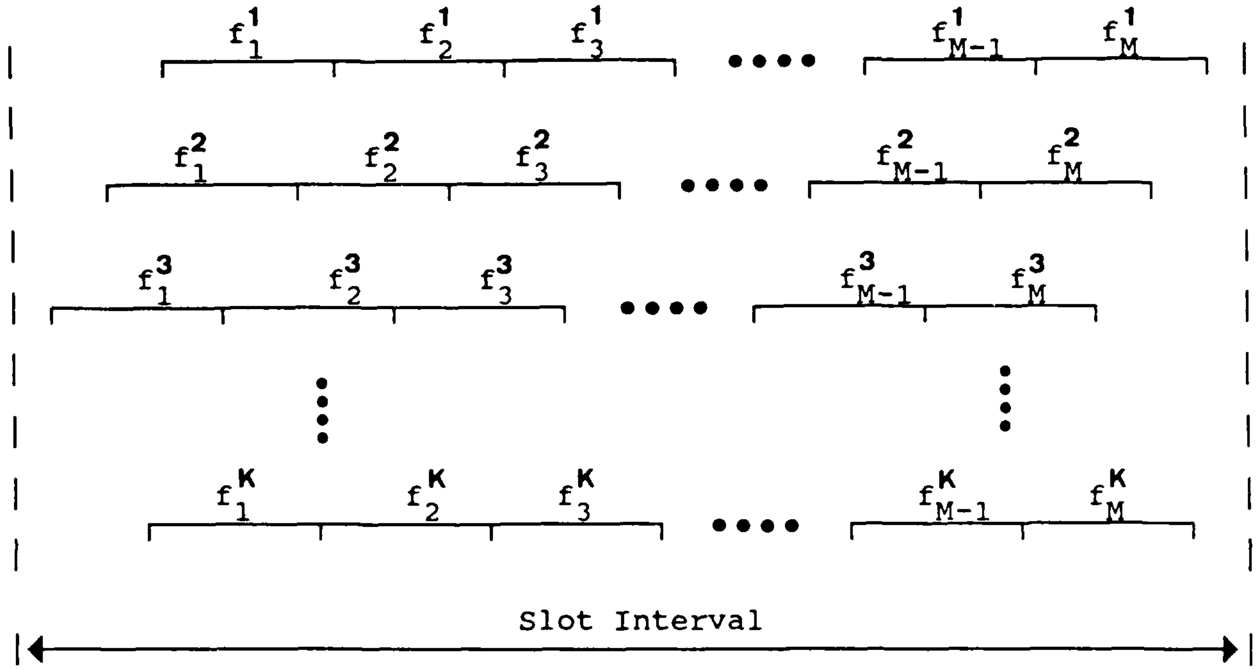

f_1^1 f_2^1 f_3^1 •••• f_{M-1}^1 f_M^1
f_1^2 f_2^2 f_3^2 •••• f_{M-1}^2 f_M^2
f_1^3 f_2^3 f_3^3 •••• f_{M-1}^3 f_M^3
f_1^K f_2^K f_3^K •••• f_{M-1}^K f_M^K
Slot Interval

Figure 1

Table 1

(31,7) RS code

q	$p_e(2)$	$\tilde{p}_e(2)$	$\lvert p_e(2)-\tilde{p}_e(2)\rvert/\tilde{p}_e(2)$
10	0.78507246	0.77127123	0.17894133D-01
25	0.93451075D-01	0.97636106D-01	0.42863559D-01
50	0.66080673D-02	0.72908919D-02	0.93654467D-01

(31,15) RS code

q	$p_e(2)$	$\tilde{p}_e(2)$	$\lvert p_e(2)-\tilde{p}_e(2)\rvert/\tilde{p}_e(2)$
10	0.13915048	0.15076248	0.77021816D-01
25	0.36450112D-03	0.53081917D-03	0.31332336
50	0.14188960D-05	0.23669732D-05	0.40054412

(31,23) RS code

q	$p_e(2)$	$\tilde{p}_e(2)$	$\lvert p_e(2)-\tilde{p}_e(2)\rvert/\tilde{p}_e(2)$
10	0.25728328D-02	0.43874633D-02	0.41359445
25	0.9370620D-07	0.28409390D-06	0.67015764

Table 1 (continued)

(63,15) RS code

q	$p_e(2)$	$\tilde{p}_e(2)$	$\lvert p_e(2)-\tilde{p}_e(2)\rvert/\tilde{p}_e(2)$
10	0.91745546	0.90631180	0.12295613D-01
25	0.58637826D-01	0.62530221D-01	0.62248220D-01
50	0.74395697D-03	0.87602105D-03	0.15075445

(63,31) RS code

q	$p_e(2)$	$\tilde{p}_e(2)$	$\lvert p_e(2)-\tilde{p}_e(2)\rvert/\tilde{p}_e(2)$
10	0.10016353	0.11197709	0.10549979
25	0.32382994D-05	0.62966015D-05	0.48570678
50	0.11704D-09	0.29755D-09	0.60665434

(63,47) RS code

q	$p_e(2)$	$\tilde{p}_e(2)$	$\lvert p_e(2)-\tilde{p}_e(2)\rvert/\tilde{p}_e(2)$
10	0.10541485D-03	0.26357027D-03	0.6000503
25	0.58D-12	0.445D-11	0.86966292

Table 1 (continued)

(31,15) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$\|p_e(3)-\tilde{p}_e(3)\|/\tilde{p}_e(3)$
10	0.85167393	0.84020289	0.13652702D-01
25	0.35959153D-01	0.39002482D-01	0.78029111D-01
50	0.38309009D-03	0.45761967D-03	0.16286358

(31,23) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$\|p_e(3)-\tilde{p}_e(3)\|/\tilde{p}_e(3)$
10	0.29646470	0.30376087	0.24019453D-01
25	0.25272762D-03	0.35074331D-03	0.27945134
50	0.13704329D-06	0.22482071D-06	0.39043298

(63,31) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$\|p_e(3)-\tilde{p}_e(3)\|/\tilde{p}_e(3)$
10	0.95754921	0.95027964	0.76499271D-01
25	0.10935348D-01	0.12442345D-01	0.12111840
50	0.35134889D-05	0.48095548D-05	0.26947731

(63,47) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$\|p_e(3)-\tilde{p}_e(3)\|/\tilde{p}_e(3)$
10	0.30594667	0.31322755	0.23244998D-01
25	0.14916148D-05	0.26688242D-05	0.44109664
50	0.115D-11	0.286D-11	0.59790209

DISTRIBUTION LIST

Copy No.	
1 - 6	Director National Research Laboratory Washington, D.C. 20375 Attention: Code 2627
7	Dr. R. N. Madan Electronics Division, Code 1114SE Office of Naval Research 800 N. Quincy Street Arlington, VA 22217-5000
8	Mr. James G. Smith Office of Naval Research Code 1241 800 N. Quincy Street Arlington, VA 22217-5000
9	Mr. Michael Georgiopoulos EECS Department University of Central Florida Orlando, FL 32816
10	Professor Mike Athans MIT, Bldg. 35 Cambridge, MA 02139
11	Professor A. Makowski Electrical Engineering Dept. University of Maryland College Park, MD 20742
12	Dr. Dave Castanon ALPHATECH, Inc. 2 Burlington Executive Center 111 Middlesex Turnpike Burlington, MA 01803-4901
13	Dr. John P. Lehoczky Dept. of Statistics Carnegie Mellon University Schenley Park Pittsburgh, PA 15213-3890
14	Professor Alex Levis MIT, Bldg. 35 Cambridge, MA 02139
15 - 26	Defense Technical Information Center, S47031 Bldg. 5, Cameron Station Alexandria, VA 22314

(Continued On Next Page)

27 - 28	P. Kazakos, EE
29	R. J. Mattauch, EE
30 - 31	E. H. Pancake, Clark Hall
32	SEAS Publications Files
33*	Office of Naval Research Resident Representative 818 Connecticut Ave., N.W. Eighth Floor Washington, D.C. 20006 Attention: Mr. Michael McCracken Administrative Contracting Officer

*Send Cover Letter Only

0333:ald:270R

UNIVERSITY OF VIRGINIA
School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,500 students with a graduate enrollment of approximately 560. There are 150 faculty members, a majority of whom conduct research in addition to teaching.

Research is a vital part of the educational program and interests parallel academic specialties. These range from the classical engineering disciplines of Chemical, Civil, Electrical, and Mechanical and Aerospace to newer, more specialized fields of Biomedical Engineering, Systems Engineering, Materials Science, Nuclear Engineering and Engineering Physics, Applied Mathematics and Computer Science. Within these disciplines there are well equipped laboratories for conducting highly specialized research. All departments offer the doctorate; Biomedical and Materials Science grant only graduate degrees. In addition, courses in the humanities are offered within the School.

The University of Virginia (which includes approximately 2,000 faculty and a total of full-time student enrollment of about 16,400), also offers professional degrees under the schools of Architecture, Law, Medicine, Nursing, Commerce, Business Administration, and Education. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. The School of Engineering and Applied Science is an integral part of this University community which provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.

END
12-87
DTIC